

Non-Fraunhofer patterns of the anharmonic Josephson current influenced by a strong interfacial pair breaking

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In the junctions with a strong Josephson coupling and a pronounced interfacial pair breaking, the magnetic interference patterns of the Josephson current are shown to differ substantially from the standard Fraunhofer shape. The Fraunhofer pattern occurs, when Josephson couplings are weak. The narrow peak of the critical current, centered at the zero magnetic field, and the suppressed hills at finite field values are the characteristic features of the non-Fraunhofer magnetic field modulation of the critical current, obtained in this paper.

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The Josephson critical current, as a function of the applied magnetic field, manifests a characteristic modulation, which originates from the fundamental quantum coherence of the superconducting leads. Under standard conditions the modulation is described by the Fraunhofer diffraction pattern, which has represented a benchmark for the magnetic field modulation of the critical current since the early days of the discovery of the Josephson effect. [1–4] Distortions of the Fraunhofer shape in tunnel junctions stem from inhomogeneities or from variations of geometry of the fabricated interfaces. [3, 4] In contemporary studies of the magnetic field induced interference patterns, single junctions are usually fabricated as a sequence of different regions, e.g., of 0 and π regions. In such junctions the patterns can substantially deviate from the standard shape, due to their phase sensitivity. The corresponding measurements constitute the basis of the Josephson interferometry method [5–18], which has been successfully used to identify symmetries of, e.g., the d -wave order parameters of cuprate superconductors [5–7], the complex p -wave superconductivity in Sr_2RuO_4 [12, 13] and the E_{2u} pairing of the superconducting UPT_3 [14]. Single Josephson junctions with inhomogeneous magnetic interlayers, fabricated as a sequence of 0 and π regions, also exhibit the interference patterns, which qualitatively differ from the Fraunhofer ones. [15–18]

This paper theoretically studies single junctions with homogeneous thin interfaces and focuses on the effects of the anharmonic phase dependence of the Josephson current on the magnetic field induced modulation. The modulation of the Josephson current will be described within the Ginzburg-Landau (GL) theory. The effects will be shown to become pronounced in the junctions with a strong Josephson coupling, where the modulation differs heavily from the Fraunhofer diffraction pattern. The results obtained could be, in particular, implemented in the junctions involving unconventional superconductors and/or magnetic interfaces. An intensive interfacial pair breaking is crucial for the systems in question. It substantially suppresses the critical current, therefore main-

taining the planar junctions with a pronounced Josephson coupling as the weak links with strongly anharmonic current-phase relations. [19]

Assume the usual form of the GL free energy, which applies, for example, to s -wave and $d_{x^2-y^2}$ -wave junctions. Let the symmetric junctions have a spatially constant width much less than the Josephson penetration length. The superconducting electrodes S_l and S_r are supposed to be thick compared to the magnetic penetration depth λ , while a homogeneous plane rectangular interlayer at $x = 0$ is assumed to be of zero length within the GL approach (see Fig. 1). The interface contribution to the GL free energy incorporates both the Josephson coupling of the two superconducting banks $g_J|\Psi_+ - \Psi_-|^2$ and the term $g(|\Psi_+|^2 + |\Psi_-|^2)$, which is responsible for the interfacial pair breaking or pair producing. It will be convenient to use the dimensionless Josephson coupling constant $g_\ell = g_J\xi(T)/K$, the interface pair breaking parameter $g_\delta = g\xi(T)/K$ and their combination $g_b(\chi) = (g_\delta + 2g_\ell \sin^2 \frac{\chi}{2})$, where $\xi(T)$ is the temperature dependent superconducting coherence length, K comes from the bulk gradient term $K|\nabla\Psi|^2$ and χ is the phase difference of the order parameters across the interface.

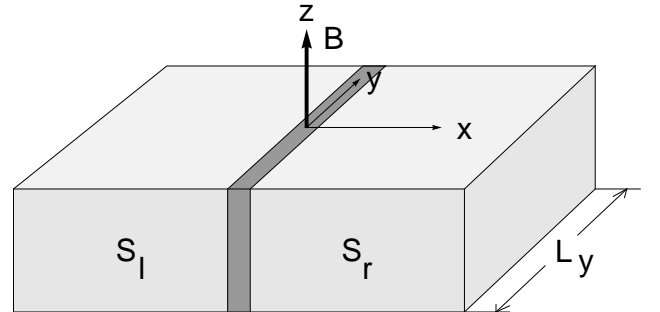


FIG. 1. Schematic diagram of the junction.

Microscopic estimations show that the range of variations of the parameters g_ℓ and g_δ (and, therefore, of g_b) is quite wide and, in particular, includes large values.

[19, 20] The limit $g_\delta \rightarrow +\infty$ corresponds to the order parameter vanishing at the boundary. In highly transparent junctions ($(1 - \mathcal{D}) \ll 1$) the parameter $g_\ell \propto (1 - \mathcal{D})^{-1}$ can be arbitrarily large. Here \mathcal{D} is the transmission coefficient. For small and moderate transparencies in dirty s -wave junctions g_ℓ can vary from vanishingly small values in the tunneling limit to those well exceeding 100 near T_c . For the pair breaking surfaces ($g_b > 0$) arbitrarily large values of g_b are admissible in the GL theory, since they correspond to spatial variations of the order parameter on a scale $\gtrsim \xi(T)$. This is in contrast to the pair producing interfaces $g_b < 0$, where the superconductivity is locally enhanced (see, e.g., [20–25]). Near the boundary a strong enhancement can induce the scale substantially less than the coherence length of the leads.

The self-consistent description of the Josephson current in planar junctions, recently developed within the GL theory in Ref. 19, takes into account the pair breaking effects produced by the phase-dependent Josephson coupling as well as by the current and by the interface itself. In particular, the following anharmonic current-phase relation has been obtained in the absence of the applied magnetic field:

$$j(g_\ell, g_\delta, \chi) = \frac{3\sqrt{3}g_\ell \sin \chi}{2(1 + 2g_\ell^2 \sin^2 \chi)} \left[1 + g_b^2(\chi) + g_\ell^2 \sin^2 \chi - \sqrt{(g_b^2(\chi) + g_\ell^2 \sin^2 \chi)^2 + 2g_b^2(\chi)} \right] j_{dp}. \quad (1)$$

Here j_{dp} is the depairing current in the bulk.

Equation (1) describes the current behavior almost perfectly, if $j < 0.7j_{dp}$. This concerns, in particular, the current at $g_\ell < 1$ for any g_δ , or at $g_\delta > 1$ for any g_ℓ . For $j > 0.7j_{dp}$ equation (1) reasonably approximates the exact numerical solution, with the deviations not exceeding 10%. [19] The solution (1) applies only to the pair breaking interfaces, i.e., to $g_b(\chi) \geq 0$. Otherwise, the parameters in equation (1) can take arbitrary values. In order to describe the current flowing through the pair producing interface, one should change the sign before the square root in equation (1). Depending on g_ℓ and g_δ , such current could exceed j_{dp} and, hence, destroy the superconductivity in the bulk of the leads. From now on only the pair breaking interfaces with $g_\ell, g_\delta > 0$ will be considered.

For a pronounced interfacial pair breaking $g_\delta^2 \gg 1$ expression (1) for the supercurrent is simplified and reduced to

$$j \approx \frac{3\sqrt{3}g_\ell \sin \chi}{4[g_\delta^2 + 4(g_\delta + g_\ell)g_\ell \sin^2 \frac{\chi}{2}]} j_{dp}. \quad (2)$$

The corresponding critical current $j_c = [3\sqrt{3}g_\ell/4g_\delta(g_\delta + 2g_\ell)]j_{dp} \ll j_{dp}$ is always small, at arbitrary g_ℓ . The validity of the condition $j \ll j_{dp}$ is the evidence of a weak link. Strongly anharmonic current-phase relation shows

up in (2) for $g_\ell^2 \gg g_\delta^2 \gg 1$, while in the case $g_\ell \ll g_\delta$ the first harmonic dominates the current.

Now let the magnetic field be applied to the junction along the z axis: $\mathbf{B}(x) = B(x)\mathbf{e}_z$ (see Fig. 1). It is assumed that the field is not too strong, substantially less than the critical fields of the leads. The well known gauge invariant result states that the supercurrent density in the presence of the field is described by its zero-field expression $j(\chi)$, if the spatially dependent phase difference across the interface $\tilde{\chi}(y) = \chi + 2\pi(y/L_y)(\Phi/\Phi_0)$, influenced by the magnetic flux Φ through the junction, is substituted for χ . [3, 4] Here $\Phi_0 = \pi\hbar c/|e|$ is the superconducting flux quantum. Considering the total supercurrent through the junction is of interest, the spatially modulated supercurrent density should be averaged over the interface area: $I = \frac{1}{L_y} \int_{-\frac{L_y}{2}}^{\frac{L_y}{2}} j\left(\chi + \frac{2\pi y \Phi}{L_y \Phi_0}\right) dy = \frac{\Phi_0}{2\pi\Phi} \int_{\varphi_-}^{\varphi_+} j(\varphi) d\varphi$. Here a rectangular plane interface is supposed to occupy the space $(-L_y/2, L_y/2)$ along the y axis. The integration limits φ_\pm are defined as $\varphi_\pm = \chi \pm \pi(\Phi/\Phi_0)$.

A macroscopic scale of the modulation period $L_y^B = \pi\ell_B^2/2\lambda = \Phi_0/2B(0)\lambda$ allows one to consider, for a given y , local densities per unit interface area of thermodynamic potential and of the supercurrent, which satisfy the relation $j(\chi) = \frac{2|e|}{\hbar} \frac{d}{d\chi} \Omega_0(\chi)$. Here $\ell_B = (\hbar c/|eB(0)|)^{1/2}$ is the magnetic length. With this relation, the Josephson current averaged over the interface is [26]

$$I = \frac{|e|\Phi_0}{\pi\Phi\hbar} \left[\Omega_0\left(\chi + \frac{\pi\Phi}{\Phi_0}\right) - \Omega_0\left(\chi - \frac{\pi\Phi}{\Phi_0}\right) \right]. \quad (3)$$

Thus, the phase dependent thermodynamic potential of the junction in the absence of the field also describes the magnetic interference pattern in the junction.

Equation (3) will be now used to establish the modulation of the anharmonic Josephson current by the applied magnetic field. Interestingly, for a strong interfacial pair breaking $g_\delta^2 \gg 1$, when the supercurrent in the absence of the field is described by (2), the averaged anharmonic current allows not only a numerical study but also an analytical description. Indeed, integrating (2) over the phase difference and using (3), one obtains the magnetic field dependent current-phase relation for the averaged anharmonic supercurrent:

$$I(\chi, \Phi) = j_{dp} \frac{\Phi_0}{\pi\Phi} \frac{3\sqrt{3}}{16(g_\delta + g_\ell)} \times \ln \left[1 + \frac{2 \sin \chi \sin \left(\frac{\pi\Phi}{\Phi_0} \right)}{\frac{g_\delta^2}{2(g_\delta + g_\ell)g_\ell} + 2 \sin^2 \left(\frac{\chi}{2} - \frac{\pi\Phi}{2\Phi_0} \right)} \right]. \quad (4)$$

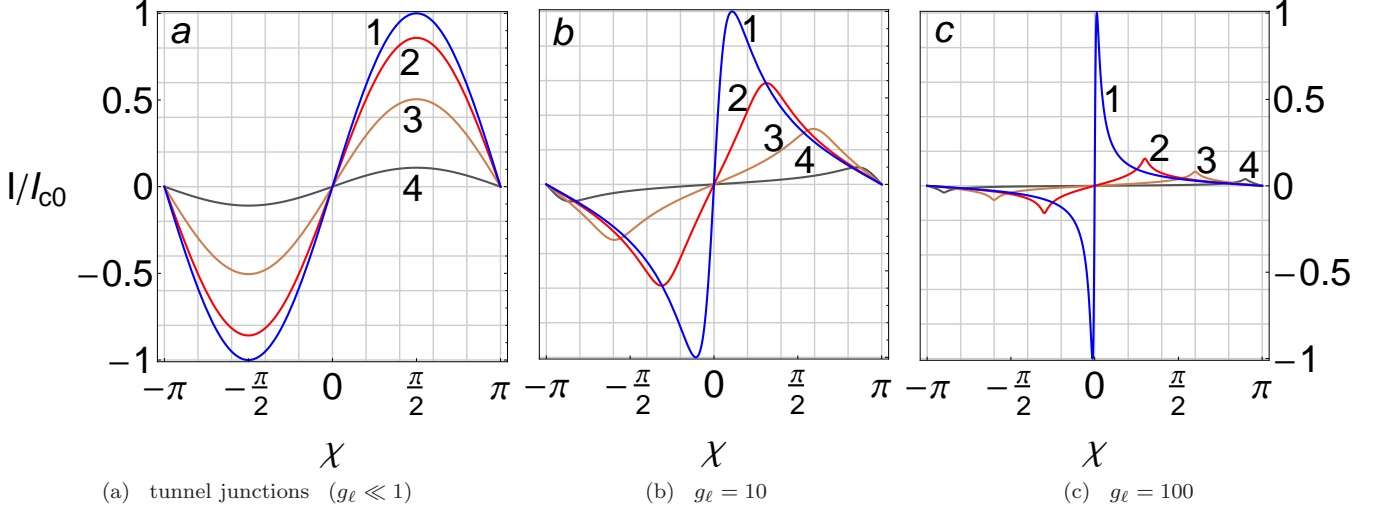


FIG. 2. The current-phase relations in junctions with the pair breaking parameter $g_\delta = 4$, taken at several values of the magnetic flux Φ : (1) $\Phi = 0$ (2) $\Phi = 0.3\Phi_0$ (3) $\Phi = 0.6\Phi_0$ (4) $\Phi = 0.9\Phi_0$. Here $\Phi_0 = \pi\hbar c/|e|$ is the flux quantum. Different panels correspond to different Josephson coupling constants. For each coupling constant the current is normalized to its critical value in the absence of the field.

The corresponding critical current is

$$I_c = \frac{3\sqrt{3}j_{dp}\Phi_0}{16\pi|\Phi|(g_\delta + g_\ell)} \ln \left[1 + \frac{2A}{\sqrt{A^2 + g_\delta^2(g_\delta + 2g_\ell)^2 - A^2}} \right], \quad (5)$$

where $A = 2(g_\delta + g_\ell)g_\ell \left| \sin\left(\frac{\pi\Phi}{\Phi_0}\right) \right|$.

Equations (4) and (5), as well as the plots shown in Figs. 2, 3 and 4 represent the central results of this paper. Fig. 2 displays the current-phase relations obtained numerically with the exact self-consistent calculations within the GL theory for $g_\delta = 4$ and for various values of g_ℓ and Φ . The corresponding equations used for this purpose will be outlined in the end of the paper. The approximate results based either on equations (1) and (3), or on the analytical formula (4), all lead to the curves, which are in excellent agreement with and practically indistinguishable from the exact ones shown in Fig. 2. Similarly, the analytical expression (5) agrees well with the numerical results for the magnetic interference patterns shown in Fig. 3.

The modulations obtained substantially differ from the Fraunhofer pattern in the junctions with the strongly anharmonic current-phase dependence, which takes place for large Josephson couplings $g_\ell^2 \gg g_\delta^2 \gg 1$. Figs. 2(b), 2(c) demonstrate that in such junctions the magnetic field substantially modifies the anharmonic structure of the current-phase relations. This is in contrast to tunnel junctions (see Fig. 2(a)), where the sinusoidal shape of the phase dependence is not distorted by the magnetic field.

An important feature of the non-Fraunhofer patterns,

obtained under the conditions $g_\ell \gg g_\delta$, $g_\delta^2 \gg 1$, is that the zero-field peak gets narrower with increasing g_ℓ . As seen in Fig. 4 and as follows from equation (5), the half width at the half of the peak is small for large values of g_ℓ/g_δ and can be approximated as $(\Delta\Phi/\Phi_0) \approx$

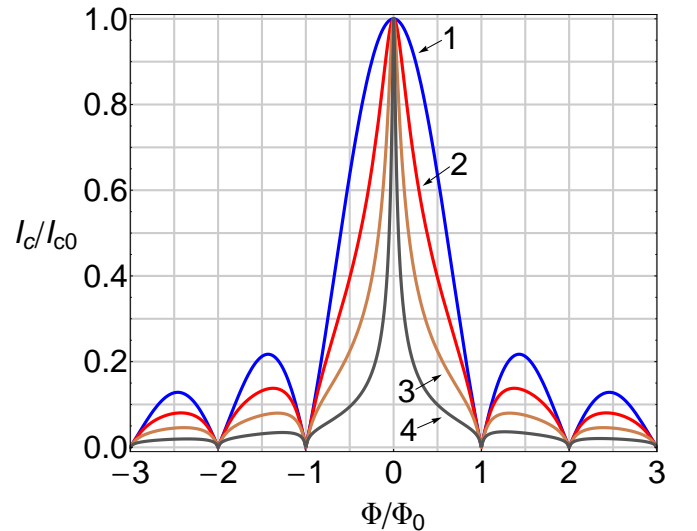


FIG. 3. The normalized critical current I_c/I_{c0} as a function of the magnetic flux, which pierces the junctions with different Josephson coupling constants: (1) small Josephson couplings $g_\ell \ll 1$, described by the Fraunhofer pattern (2) $g_\ell = 10$ (3) $g_\ell = 30$ (4) $g_\ell = 100$. The interfacial pair breaking parameter is $g_\delta = 4$; I_{c0} is the zero-field critical current for the corresponding junctions. The curves have been plotted based on equations (1) and (3).

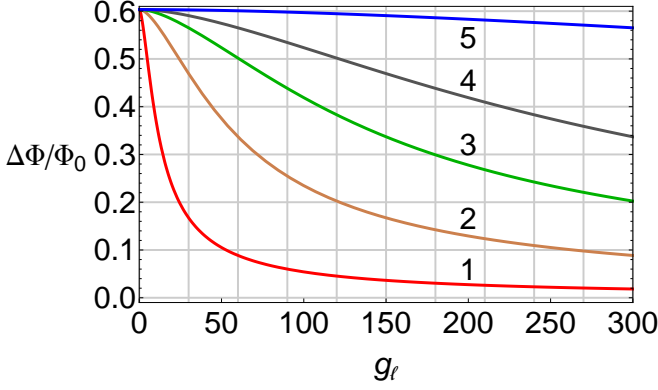


FIG. 4. The half width at the half of the zero-field peak as a function of the Josephson coupling constant g_ℓ , according to equation (5). The curves are plotted for various values of the interfacial pair breaking parameter g_δ : (1) $g_\delta = 4$ (2) $g_\delta = 20$ (3) $g_\delta = 50$ (4) $g_\delta = 100$ (5) $g_\delta = 500$.

$1.35g_\delta/g_\ell \ll 1$. In the opposite case $g_\ell/g_\delta \ll 1$ the quantity $(\Delta\Phi/\Phi_0)$ is close to its Fraunhofer constant value ≈ 0.6 and weakly depends on g_ℓ . Also, the hills at the finite fields shown in Fig. 3 are significantly suppressed as compared to the Fraunhofer ones, in measure of the same small parameter $g_\delta/g_\ell \ll 1$ and up to a logarithmic factor. The larger the Josephson coupling, the stronger the suppression of the critical current by the magnetic field within each interval $n\Phi_0 < \Phi < (n+1)\Phi_0$ ($n = 0, \pm 1, \pm 2, \dots$). The Fraunhofer pattern is represented by the curve 1 in Fig. 3. It appears for $g_\ell \ll g_\delta$, when the first harmonic dominates the current. Then one gets from (5) $I_c(\Phi) = I_{c0} |\sin(\pi\Phi/\Phi_0)| / (\pi|\Phi|/\Phi_0)$, where $I_{c0} = j_{dp}3\sqrt{3}g_\ell/4g_\delta^2$ is the corresponding critical current in the absence of the magnetic field.

Let us return now to a brief description of the approach used for plotting Fig. 2 and based on the results of Ref. [19]. For the order parameter $f(x)e^{i\chi(x)}$ normalized to $f = 1$ in the bulk without superflow, one writes the first integral of the GL equation in the presence of the supercurrent [27]

$$\left(\frac{df}{d\tilde{x}}\right)^2 + f^2 - \frac{1}{2}f^4 + \frac{4\tilde{j}^2}{27f^2} = 2f_\infty^2 - \frac{3}{2}f_\infty^4. \quad (6)$$

Here $\tilde{x} = x/\xi(T)$, $\tilde{j} = j/j_{dp} = -(3\sqrt{3}/2)(d\chi/d\tilde{x})f^2$ and f_∞ is the asymptotic value of f in the depth of the leads.

The boundary conditions for f and the expression for the supercurrent in the symmetric junctions are

$$\left(\frac{df}{d\tilde{x}}\right)_\pm = \pm g_b(\chi)f_0, \quad \tilde{j} = \frac{3\sqrt{3}}{2}g_\ell f_0^2 \sin \chi. \quad (7)$$

One puts $x = 0$ in (6) and, using (7), eliminates the current and the first derivative of the order parameter. This results in a biquadratic relation between f_0^2 and

f_∞^2 . Then, using the asymptotic formulas in the bulk $\tilde{j} = (3\sqrt{3}/2)\tilde{v}_s(1 - \tilde{v}_s^2)$, $f_\infty^2 = 1 - \tilde{v}_s^2$ [4] and equating the current to that in (7) with $f_0^2 = (1 - \tilde{v}_s^2)\alpha$, one obtains $\tilde{v}_s = \alpha g_\ell \sin \chi$. As \tilde{v}_s , f_0 , f_∞ and \tilde{j} are now expressed via the only variable α , one gets from the biquadratic relation the fourth-order polynomial equation for α [19]

$$2g_b^2(\chi)\alpha - (1 - \alpha)^2[1 - \alpha(\alpha + 2)g_\ell^2 \sin^2 \chi] = 0. \quad (8)$$

Eq. (8) is exact within the conventional GL approach with the boundary conditions (7). Calculating $\alpha(g_\ell, g_\delta, \chi)$, one gets $j(\chi)$ and finds numerically $\Omega_0(\chi)$. Then, from equation (3) with the calculated $\Omega_0(\chi)$, one obtains the self-consistent current-phase relations shown in Fig. 2.

In conclusion, it is demonstrated that the anharmonic phase dependence of the Josephson current, taking place in the junctions with a strong Josephson coupling and a pronounced interfacial pair breaking, has a profound influence on the phase sensitive magnetic interference patterns. The distinctive features of the non-Fraunhofer patterns uncovered, are the narrow peak of the critical current, centered at the zero field, and the suppressed hills at finite field values. The results obtained could be implemented in the junctions involving unconventional superconductors and/or magnetic interfaces.

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